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Analytical, numerical solutions and study of stability, resonance for nonlinear vibro-impact system under different excitation

M.Sayed [1, 2] & Y. S. Hamed [1, 2]

Abstract—Because of the existence of impacts, the vibro-impact system is discontinuous and strongly nonlinear, such as hammer-like devices, rotor-casing dynamical systems, heat exchangers, fuel elements of nuclear reactor, gears, piping systems, wheel–rail interaction of high speed railway coaches. Researches into the dynamic behavior of vibro-impact systems have important significance in optimization design of machinery and noise suppression. Hence, the complication of the dynamics of vibro-impact system has received great attention. In this paper, the analytical solution of non-linear vibro-impact system using multiple time scale method up to and including second order approximations is obtained. All resonance cases from mathematical solution are extracted. Also, the numerical solution of non-linear vibro-impact system using Runge-Kutta method of order four are obtained. The stability of the non-linear vibro-impact system at the worst resonance case is studied. The behaviors of the system at different values of parametric excitation are investigated. The effects of various parameters on the behavior of the system are studied. A comparison with the available published work is reported.

Index Terms— Vibro-impact system, Multiple time scale, Vibrations, Resonance, Stability.

1 INTRODUCTION

Because of the existence of impacts, the vibro-impact system is discontinuous and strongly nonlinear, such as hammer-like devices, rotor-casing dynamical systems, heat exchangers, fuel elements of nuclear reactor, gears, piping systems, wheel–rail interaction of high speed railway coaches. Researches into the dynamic behavior of vibro-impact systems have important significance in optimization design of machinery and noise suppression. Hence, the complication of the dynamics of vibro-impact system has received great attention.

Budd and Dux [1] proved that the periodic motion of the single-degree-of-freedom vibro-impact system cannot have Hopf bifurcation. In recent years, many researchers investigated some two- and three-degree of freedom (3-dof) of vibro-impact systems, and found that these vibro-impact systems can exhibit rich dynamic behavior, and have various bifurcations, such as period-doubling bifurcation [2, 3], Hopf bifurcation [4, 5]. Besides, there are some studies on calculation of Lyapunov exponents [6, 7], controlling chaos [8, 9] and rising phenomena and the multi-sliding bifurcation [10] in systems with impacts. Dynamics of vibro-impact system in two cases of resonance (1:3 and 1:4 resonance) was also studied by Ding and Xie [11]. Xie and Ding [12] considered Hopf–Hopf bifurcation of a 3-dof vibro-impact system. When two pairs of complex conjugate eigenvalues of the Jacobian matrix of the map at fixed point cross the unit circle simultaneously, the six-dimensional Poincare´ map was reduced to its four-dimensional normal form by the center manifold and the normal form methods. It was shown that there are torus T1 and T2 bifurcation under some parameter combinations. In Ref. [13], we considered a two-degree-of-freedom (2-dof) vibro-impact system with symmetric rigid constraints, and described the symmetry of Poincare´ map. It was shown that if the Jacobian matrix of the Poincare´ map at the fixed point has a real eigenvalue crossing the unit circle at +1, the symmetric fixed point will bifurcate into two anti-symmetric fixed points which have the same stability via pitch fork bifurcation. In [14], expanded the symmetry of Poincare´ map of the 2-dof vibro-impact system discussed in Ref. [13] to a 3-dof vibro-impact system with symmetric rigid constraints, and paid more attention to the effect of the symmetry of Poincare´ map on possible bifurcations.

Sayed and Mousa [15] investigated the influence of the quadratic and cubic terms on non-linear dynamic characteristics of the angle-ply composite laminated rectangular plate with parametric and external excitations. The method of multiple time scale perturbation is applied to solve the non-linear differential equations describing the system up to and including the second-order approximation. Two cases of the sub-harmonic resonances cases ($\Omega_2 \cong 2\omega_1$ and $\Omega_2 \cong 2\omega_2$) in the presence of 1:2 internal resonance $\omega_2 \cong 2\omega_1$ are considered. The stability of the system is investigated using both frequency response equations and phase-plane method. It is quite clear that some of the simultaneous resonance cases are undesirable in the design of such system as they represent some of the worst behavior of the system. Such cases should be avoided as working

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conditions for the system. Sayed and Mousa [16] studied an analytical investigation of the nonlinear vibration of a symmetric cross-ply composite laminated piezoelectric rectangular plate under parametric and external excitations. Their study focused on the case of 1:1:3 primary resonances and internal resonance, and they verified the analytical results calculated by the method of multiple time scale by comparing them with the numerical results of the modal equations. The obtained results were verified by comparing the results of the finite difference method (FDM) and Runge-Kutta (RKM) method. Eissa and Sayed [17-19] and Sayed [20], studied the effects of different active controllers on simple and spring pendulum at the primary resonance via negative velocity feedback or its square or cubic. Amer et al. [21], studied the dynamical system of a twin-tail aircraft, which is described by two coupled second order nonlinear differential equations having both quadratic and cubic nonlinearities, solved and controlled. The system is subjected to both multi-parametric and multi-external excitations. The method of multiple time scale perturbation is applied to solve the nonlinear differential equations up to the two order approximations. The stability of the system is investigated applying both frequency response equations and phase plane method. Two simple active control laws based on the linear negative velocity and acceleration feedback are used. Sayed and Hamed [22] studied the response of a two-degree-of-freedom system with quadratic coupling under parametric and harmonic excitations. The method of multiple scale perturbation technique is applied to solve the non-linear differential equations and obtain approximate solutions up to and including the second-order approximations. Sayed and Kamel [23, 24] investigated the effect of different controllers on the vibrating system and the saturation control of a linear absorber to reduce vibrations due to rotor blade flapping motion. The stability of the obtained numerical solution is investigated using both phase plane methods and frequency response equations. Sayed et al. [25] investigated the non-linear dynamics of a two-degree-of freedom vibration system including quadratic and cubic non-linearities subjected to external and parametric excitation forces. There exist multi-valued solutions which increase or decrease by the variation of some parameters. The numerical simulations show the system exhibits periodic motions and chaotic motions. Amer and Sayed [26] studied the response of one-degree-of freedom, non-linear system under multi-parametric and external excitation forces simulating the vibration of the cantilever beam. The solution of this system up to and including the second order approximation is determined applying the multiple time scale perturbation. The steady state solution and its stability are determined. Hamed et al. [27-29] studied USM model subject to multi-external or both multi-external and multi-parametric and both multi-external and tuned excitation forces. The model consists of multi-degree-of-freedom system consisting of the tool holder and absorbers (tools) simulating ultrasonic machining process. The advantages of using multi-tools are to machine different materials and different shapes at the same time. This leads to time saving and higher machining efficiency. Kamel and Hamed [30] studied the nonlinear behavior of an inclined cable subjected to harmonic excitation near the simultaneous primary and 1:1 internal resonance using multiple scale method. Hamed et al. [31] presented the behavior of the nonlinear string beam coupled system subjected to external, parametric and tuned excitations for case 1:1 internal resonance. The stability of the system studied using frequency response equations and phase-plane method. It is found from numerical simulations that there are obvious jumping phenomena in the frequency response curves. Kamel et al. [32] studied a model subject to multi-external excitation forces. The model is represented by two-degree-of-freedom system consisting of the main system and absorber simulating ultrasonic machining. They used the passive vibration controller to suppress the vibration behavior of the system.

2. Mathematical Modeling

A system of three-degree of freedom with symmetric rigid constraints is shown in Fig. 1 [14]. The system has three masses M_1, M_2 and M_3. M_2 and M_3 are connected to rigid planes via two linear springs K_2 and K_3, and two linear viscous dashpots C_2 and C_3, respectively. M_1 is connected to M_2 via linear spring K_1 and linear viscous dashpot C_1. The excitations on three masses are harmonic with amplitudes P_1, P_2 and P_3. For small forcing amplitudes the system undergoes simple oscillations and behaves as a linear system. However, as the amplitudes increased, M_3 begins to collide with two stops of M_2, and the system becomes discontinuous and strongly non-linear. C_1 and C_2 are assumed as proportional damping. Between any two consecutive impacts, the non-dimensional differential equations of motion are given by

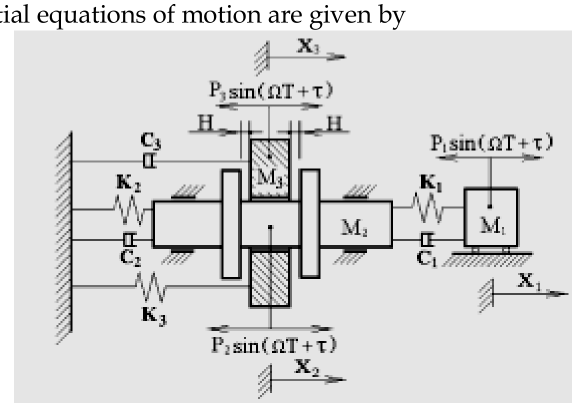

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Fig.1. A three-degree-of-freedom vibro-impact system with symmetric rigid constraints [14].

$$\ddot{X}_1 + \varepsilon\hat{c}_1(\dot{X}_1 - \dot{X}_2) + \omega_1^2(X_1 - X_2) + \varepsilon\hat{\alpha}_1(X_1 - X_2)^3 = \varepsilon X_1 \hat{F}_1 \sin(\Omega t + \tau) \quad (1)$$

$$\ddot{X}_2 + \varepsilon\hat{c}_2\dot{X}_2 + \varepsilon\hat{c}_3(\dot{X}_2 - \dot{X}_1) + \omega_2^2 X_2 + \varepsilon\hat{\beta}_1(X_2 - X_1) + \varepsilon\hat{\alpha}_2(X_2 - X_1)^3 = \varepsilon X_2 \hat{F}_2 \sin(\Omega t + \tau) \quad (2)$$

$$\ddot{X}_3 + \varepsilon\hat{c}_4\dot{X}_3 + \omega_3^2 X_3 + \varepsilon\hat{\beta}_2 X_3^3 = \varepsilon X_3 \hat{F}_3 \sin(\Omega t + \tau) \quad (3)$$

The parameters $\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4, \hat{\alpha}_1, \hat{\alpha}_2, \hat{\beta}_1, \hat{\beta}_2, \hat{F}_1$, $\hat{F}_2$ and $\hat{F}_3$ are of the order of 1.

3. PERTURBATION SOLUTION

The approximate solution of Eqs. (1)-(3) can be obtained by using the method of multiple scales [33-34]. Let

$$X_n(t;\varepsilon) = x_{n0}(T_0, T_1) + \varepsilon x_{n1}(T_0, T_1) \qquad (n = 1, 2, 3) \quad (4)$$

where, $T_n = \varepsilon^n t$ (n = 0, 1) are the fast and slow time scales respectively. In terms of T_0 and T_1, the time derivatives transform according to

$$\frac{d}{dt} \equiv D_0 + \varepsilon D_1 \ , \quad \frac{d^2}{dt^2} \equiv D_0^2 + 2\varepsilon D_0 D_1 \quad (5)$$

where $D_n = \partial/\partial T_n$. Substituting Eqs. (4)-(5) into Eqs. (1)-(3) and equating coefficients of similar powers of ε, one obtains:

Order ε^0:

$$(D_0^2 + \omega_1^2)x_{10} = \omega_1^2 x_{20} \quad (6)$$

$$(D_0^2 + \omega_2^2)x_{20} = 0 \quad (7)$$

$$(D_0^2 + \omega_3^2)x_{30} = 0 \quad (8)$$

Order ε^1:

$$(D_0^2 + \omega_1^2)x_{11} = -2D_0 D_1 x_{10} + \omega_1^2 x_{21} - \hat{c}_1(D_0 x_{10} - D_0 x_{20}) - \hat{\alpha}_1(x_{10}^3 + 3x_{10}x_{20}^2 - 3x_{10}^2 x_{20} - x_{20}^3) + x_{10}\hat{F}_1 \sin(\Omega t + \tau) \quad (9)$$

$$(D_0^2 + \omega_2^2)x_{21} = -2D_0 D_1 x_{20} - \hat{c}_2 D_0 x_{20} - \hat{c}_3(D_0 x_{20} - D_0 x_{10}) - \hat{\beta}_1(x_{20} - x_{10}) - \hat{\alpha}_2(x_{20}^3 + 3x_{20}x_{10}^2 - 3x_{20}^2 x_{10} - x_{10}^3) + x_{20}\hat{F}_2 \sin(\Omega t + \tau) \quad (10)$$

$$(D_0^2 + \omega_3^2)x_{31} = -2D_0 D_1 x_{30} - \hat{c}_4 D_0 x_{30} - \hat{\beta}_2 x_{30}^3 + x_{30}\hat{F}_3 \sin(\Omega t + \tau) \quad (11)$$

The solution of Eqs. (6)-(8) can be expressed in the complex form:

$$x_{10} = A_1 \exp(i\omega_1 T_0) + \Gamma_1 A_2 \exp(i\omega_2 T_0) + cc \quad (12)$$

$$x_{20} = A_2 \exp(i\omega_2 T_0) + cc \quad (13)$$

$$x_{30} = A_3 \exp(i\omega_3 T_0) + cc \quad (14)$$

where $\Gamma_1 = \omega_1^2/(\omega_1^2 - \omega_2^2)$ and cc denotes the complex conjugate of the preceding terms and the $A_n, (n = 1, 2, 3)$ are to be determined through the elimination of secular and small-divisor terms from the first order approximation. Substituting Eqs. (12)-(14) into Eqs. (9)-(11), eliminating the secular terms, then the first-order approximations are given by

$$\begin{aligned} x_{11}(T_0,T_1) &= E_1 \exp(i\omega_2 T_0) + E_2 \exp(3i\omega_1 T_0) + E_3 \exp(3i\omega_2 T_0) \\ &+ E_4 \exp(i(\omega_1 + 2\omega_2)T_0) + E_5 \exp(i(\omega_1 - 2\omega_2)T_0) + E_6 \exp(i(2\omega_1 + \omega_2)T_0) \\ &+ E_7 \exp(i(2\omega_1 - \omega_2)T_0) + E_8 \exp(i(\Omega + \omega_1)T_0 + i\tau) + E_9 \exp(i(\Omega - \omega_1)T_0 + i\tau) \\ &+ E_{10} \exp(i(\Omega + \omega_2)T_0 + i\tau) + E_{11} \exp(i(\Omega - \omega_2)T_0 + i\tau) + cc \end{aligned} \quad (15)$$

$$\begin{aligned} x_{21}(T_0,T_1) &= H_1 \exp(i\omega_1 T_0) + H_2 \exp(3i\omega_1 T_0) + H_3 \exp(3i\omega_2 T_0) \\ &+ H_4 \exp(i(\omega_2 + 2\omega_1)T_0) + H_5 \exp(i(\omega_2 - 2\omega_1)T_0) + H_6 \exp(i(\omega_1 - 2\omega_2)T_0) \\ &+ H_7 \exp(i(\omega_1 + 2\omega_2)T_0) + H_8 \exp(i(\Omega + \omega_2)T_0 + i\tau) + H_9 \exp(i(\Omega - \omega_2)T_0 + i\tau) + cc \end{aligned} \quad (16)$$

$$x_{31}(T_0,T_1) = G_1 \exp(3i\omega_3 T_0) + G_2 \exp(i(\Omega + \omega_3)T_0 + i\tau) + G_3 \exp(i(\Omega - \omega_3)T_0 + i\tau) + cc \quad (17)$$

where E_i $(i = 1, 2, ..., 11)$, H_j $(j = 1, 2, ..., 9)$ and G_k $(k = 1, 2, 3)$ are complex functions in T_1. From the above derived solutions, the reported resonance cases are

(A) Primary Resonance: $\Omega + \frac{1}{2}\tau \cong \omega_n$, $n = 1, 2, 3.$

(B) Sub-Harmonic Resonance:

$\Omega + \tau \cong 2\omega_n$, $\Omega + \tau \cong 4\omega_n$, $n = 1, 2, 3.$

(C) Internal or Secondary Resonance:

$\omega_1 \cong s\,\omega_2$, $\omega_2 \cong s\,\omega_1$ $s = 1, 2, 3, 5.$

(D) Combined Resonance:

$\pm\Omega + \tau \cong \pm\omega_1 \pm \omega_2$

(E) Simultaneous or Incident Resonance: Any combination of the above resonance cases is considered as simultaneous or incident resonance.

3. STABILITY OF THE SYSTEM

Using one of the worst simultaneous principle parametric resonance $\Omega \cong 2\omega_2$, $\Omega \cong 2\omega_3$ in the presence of internal resonance conditions $\omega_1 \cong 3\omega_2$ (confirmed numerically). To describe how close the frequencies are to the resonance conditions we introduce detuning parameters $\Omega = 2\omega_2 + \sigma_2$, $\Omega = 2\omega_3 + \sigma_3$ and $\omega_1 = 3\omega_2 + \sigma_1$ (where $\sigma_1 = \varepsilon\hat{\sigma}_1$ and $\sigma_2 = \varepsilon\hat{\sigma}_2$, $\sigma_3 = \varepsilon\hat{\sigma}_3$ are called internal and external detuning parameters) and eliminating the secular and small-divisor terms leads to the solvability conditions for the first approximations as:

$$2i\omega_1 D_1 A_1 = i\,\omega_1(\hat{c}_3\Gamma_1 - \hat{c}_1)A_1 + \hat{\beta}_1 A_1 + 6(\hat{\alpha}_2 - \hat{\alpha}_1)(1 - \Gamma_1)A_1 A_2 \bar{A}_2 + 3(\hat{\alpha}_2 - \hat{\alpha}_1)A_1^2 \bar{A}_1 + \left[\frac{\omega_1^2 \hat{\alpha}_2}{8\omega_2^2} + \hat{\alpha}_1\right]\left[1 - \Gamma_1\right]^3 A_2^3 \exp(-i\hat{\sigma}_1 T_1)$$

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$$-\frac{i\,\Gamma_1\hat{F}_1A_2}{2}\exp(-i\,\hat{\sigma}_1T_1+i\,\tau)-\frac{i\,\hat{F}_2\omega_1^2A_2}{2\left[\omega_2^2-(\Omega+\omega_2)^2\right]}\exp(i\,(\hat{\sigma}_2-\hat{\sigma}_1)T_1+i\,\tau) \quad (18)$$

$$2i\,\omega_2D_1A_2=-i\,\omega_2(\hat{c}_2+\hat{c}_3+\Gamma_1)A_2-\hat{\beta}_1(1-\Gamma_1)A_2-3\hat{\alpha}_2(1-\Gamma_1)^3A_2^2\bar{A}_2$$
$$-6\hat{\alpha}_2(1-\Gamma_1)A_1\bar{A}_1A_2+3\hat{\alpha}_2\bar{A}_2^2A_1\exp(i\,\hat{\sigma}_1T_1)-\frac{i\,\hat{F}_2\bar{A}_2}{2}\exp(i\,\hat{\sigma}_2T_1+i\,\tau) \quad (19)$$

$$2i\,\omega_3D_1A_3=-i\,\hat{c}_4\omega_3A_3-3\hat{\beta}_2A_3^2\bar{A}_3-\frac{i\,\hat{F}_3\bar{A}_3}{2}\exp(i\,\hat{\sigma}_3T_1+i\,\tau) \quad (20)$$

To analyze the solutions of Eqs. (18)-(20), we express A_n in the polar form

$$A_n=(a_n/2)e^{i\gamma_n}\ , \quad (n=1,2,3) \quad (21)$$

where a_n and γ_n are the steady state amplitudes and phases of the motion respectively. Substituting Eq. (21) into Eqs. (18)-(20) and equating the real and imaginary parts. Hence, the steady state solutions of equations are given by

$$\frac{(c_3\Gamma_1-c_1)}{2}a_1+\frac{\Gamma_2}{8\omega_1}a_2^3\sin\theta_1+\Gamma_3F_2a_2\cos(\theta_1-\theta_2+\tau)$$
$$-\frac{\Gamma_1F_1}{4\omega_1}a_2\cos(\theta_1-\theta_2+\tau)=0 \quad (22)$$

$$(\frac{3}{2}\sigma_2-\sigma_1)a_1+\frac{\beta_1}{2\omega_1}a_1+\frac{3(\alpha_2-\alpha_1)(1-\Gamma_1)}{4\omega_1}a_1a_2^2$$
$$+\frac{3(\alpha_2-\alpha_1)}{8\omega_1}a_1^3+\frac{\Gamma_2}{8\omega_1}a_2^3\cos\theta_1-\Gamma_3F_2a_2\sin(\theta_1-\theta_2+\tau)$$
$$+\frac{\Gamma_1F_1}{4\omega_1}a_2\sin(\theta_1-\theta_2+\tau)=0 \quad (23)$$

$$\frac{(c_2+c_3+\Gamma_1)}{2}a_2+\frac{3\alpha_2}{8\omega_2}a_1a_2^2\sin\theta_1+\frac{F_2}{4\omega_2}a_2\cos\theta_2=0 \quad (24)$$

$$\frac{1}{2}\sigma_2a_2-\frac{\beta_1(1-\Gamma_1)}{2\omega_2}a_2-\frac{3\alpha_2(1-\Gamma_1)^3}{8\omega_2}a_2^3-\frac{3\alpha_2(1-\Gamma_1)}{4\omega_2}a_1^2a_2$$
$$+\frac{3\alpha_2}{8\omega_2}a_1a_2^2\cos\theta_1-\frac{F_2}{4\omega_2}a_2\sin\theta_2=0 \quad (25)$$

$$\frac{c_4}{2}a_3+\frac{F_3}{4\omega_3}a_3\cos(\theta_3+\tau)=0 \quad (26)$$

$$\frac{1}{2}\sigma_3a_3-\frac{3\beta_2}{8\omega_3}a_3^3-\frac{F_3}{4\omega_3}a_3\sin(\theta_3+\tau)=0 \quad (27)$$

4. RESULTS AND DISCUSSIONS

The three-degree-of-freedom non-linear vibro-impact system under parametric excitations is studied. The solution of this system is determined up to the second order approximation using the multiple time scale perturbation. To study the behavior of the system of Eqs. (1-3), the Runge-Kutta of fourth order method was applied to determine the numerical solution of the given system. Fig. 2 illustrates the response for the non-resonant system where $\Omega \neq \omega_1 \neq \omega_2 \neq \omega_3$ at some values of the equation parameters . It is observed from this figure that the oscillation responses of the three modes of vibro-impact system start with increasing amplitude and the steady state amplitudes are closed to zero.

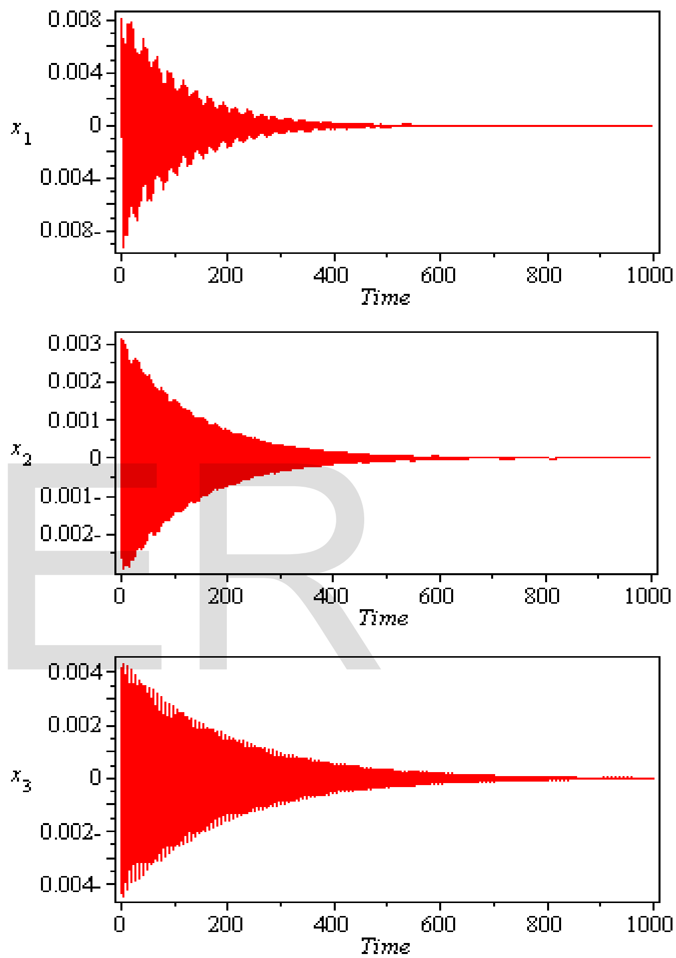

Fig. 2 Non-resonance system behavior (basic case)
C_1 = 0.02 C_2 = 0.01, C_3 = 0.002, C_4 = 0.01, α_1= 0.5, α_2= 0.05, β_1= 0.324, β_2= 0.2, $\Omega \neq \omega_1 \neq \omega_2 \neq \omega_3$, F_1 = 0.3 , F_2 = 0.4 , F_3 = 0.8

Fig. 3 shows that the time response of the simultaneous sub-harmonic and internal resonance case where ($\Omega \cong 2\omega_2$, ($\Omega \cong 2\omega_3$, $\omega_1 \cong 3\omega_2$), which is one of the worst resonance cases. It is observed from this figure that the oscillation responses of the three modes of vibro-impact system start with zero amplitude and the steady state amplitudes are increasing due to tuned oscillations. Also from this figure we have that the amplitudes are increased to about 9600%, 8750% and 200% of the maximum excitation forces amplitude F_3 respectively.

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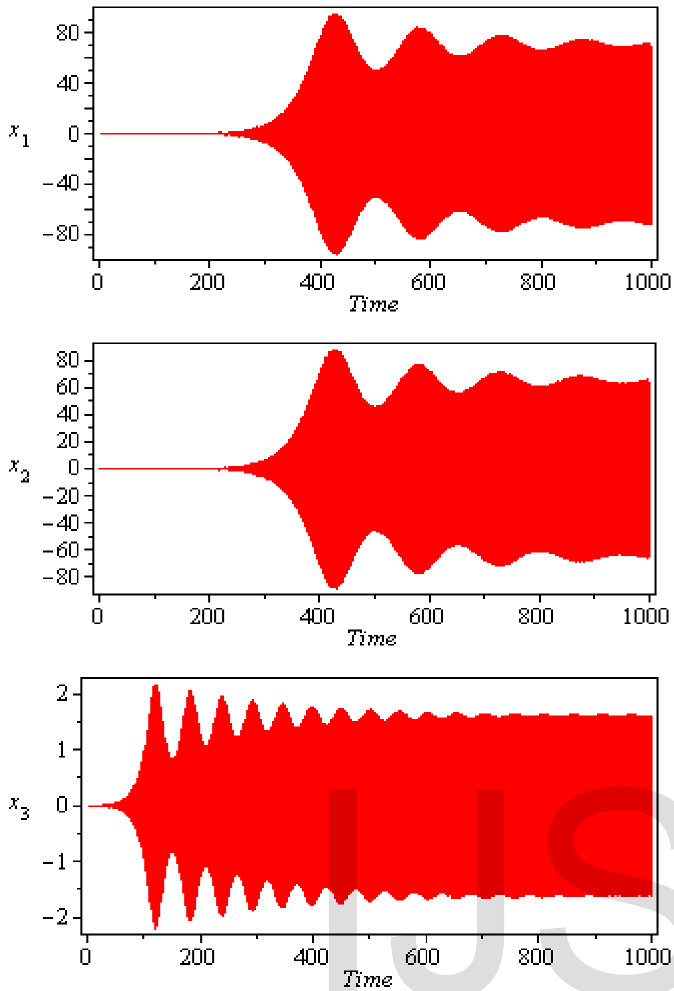

Fig. 3. Simultaneous sub-harmonic and internal resonance case ($\Omega \cong 2\omega_2$, $\Omega \cong 2\omega_3$, $\omega_1 \cong 3\omega_2$)

4.1. Response Curves and Effects of Different Parameters

In this section, the steady state response of the given system at various parameters near the simultaneous sub-harmonic and internal resonance case is investigated and studied. The frequency response equations given by Eqs. (22-27) are solved numerically at the same values of the parameters shown in Fig. 2.

Fig. 4a, show the steady state amplitudes of the first mode of vibro-impact system against the detuning parameters σ_1, the response curve is bent to the left leading to the occurrence of the jump phenomena and multi-valued amplitude.

Figs. 4b show that the steady state amplitude of the first modes of vibro-impact system is a monotonic decreasing function in the linear damping coefficients c_1. For negative and postive values of the nonlinear parameter α_1 the curves of the first modes of vibro-impact system are bent to the right and left and have harding and softing spring type and there exists jump phenomena and multi-valued amplitudes as shown in Fig. 4c. Fig. 4d shows that the steady state amplitude is a monotonic increasing function in the excitation amplitude F_1.

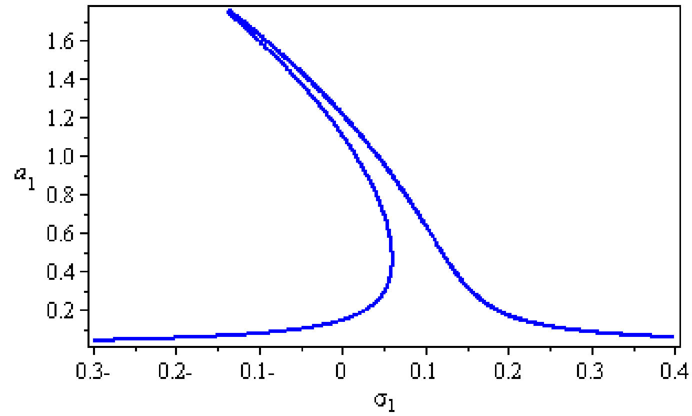

Fig.4a. Effects of the detuning parameter σ_1

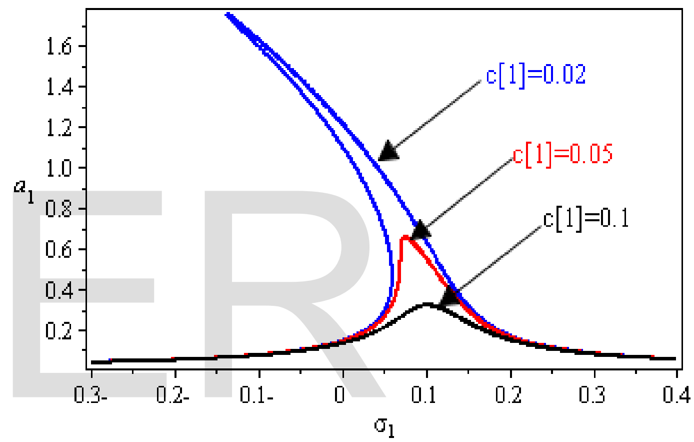

Fig.4b. Effects of the damping coefficient c_1

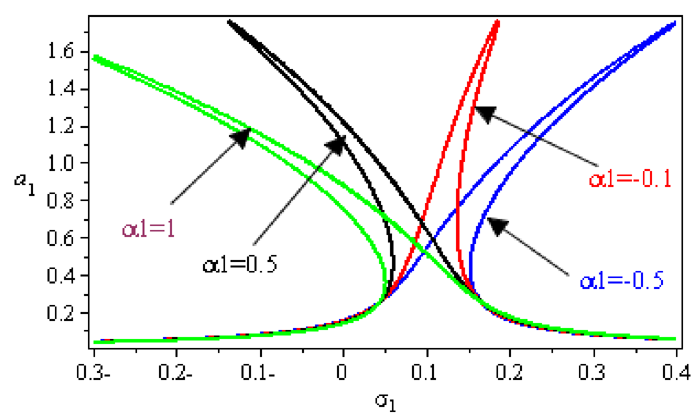

Fig.4c. Effects of the nonlinear parameter α_1

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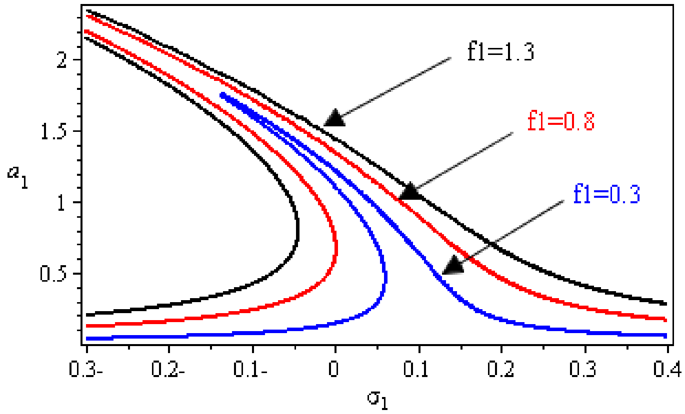

Fig.4d. Effects of the excitation amplitude F_1

Fig. 5a, show the steady state amplitudes of the second mode of vibro-impact system against the detuning parameters σ_2, the response curve is concave up. Figs. 5 (b, c) show that the steady state amplitude of the second modes of vibro-impact system is a monotonic decreasing function in the linear damping coefficients c_2 and the nonlinear parameter α_2, the response curves are concave up leading to the occurrence of the jump phenomena and multi-valued amplitude. Fig. 4d shows that the steady state amplitude is a monotonic increasing function in the excitation amplitude F_2.

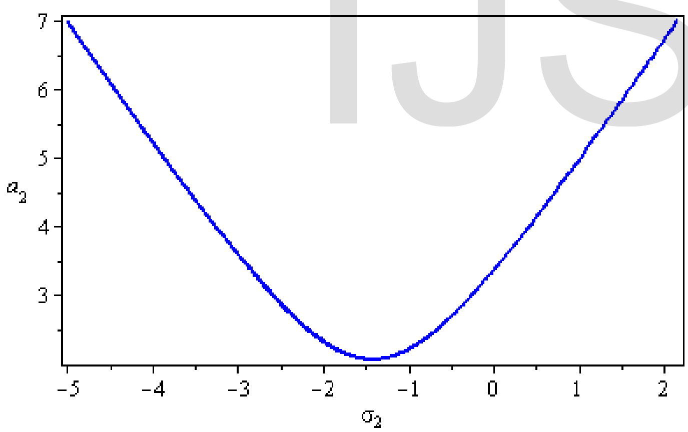

Fig.5a. Effects of the detuning parameter σ_2

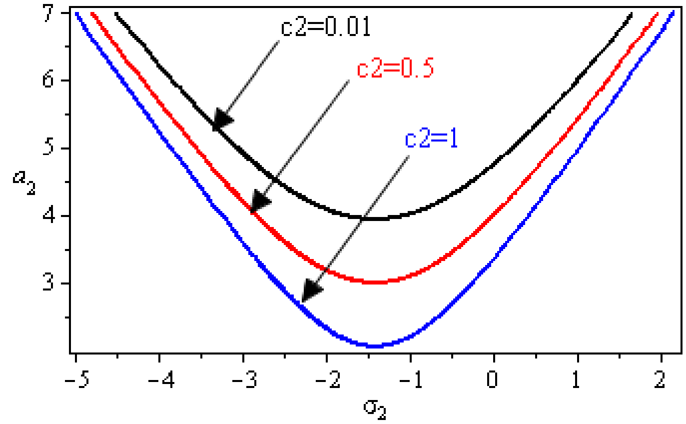

Fig.5b. Effects of damping coefficient c_2

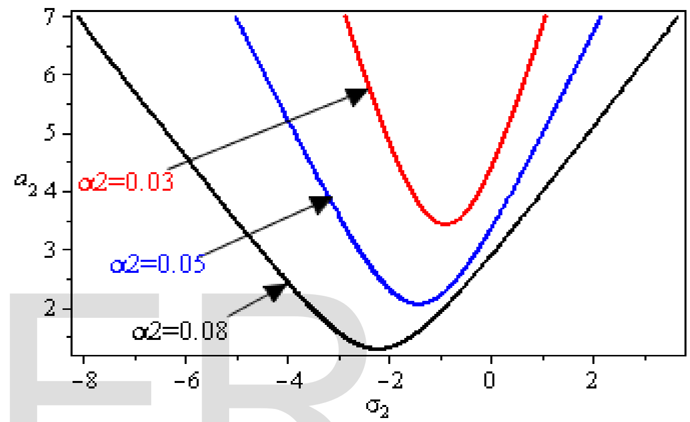

Fig.5c. Effects of the nonlinear parameter α_2

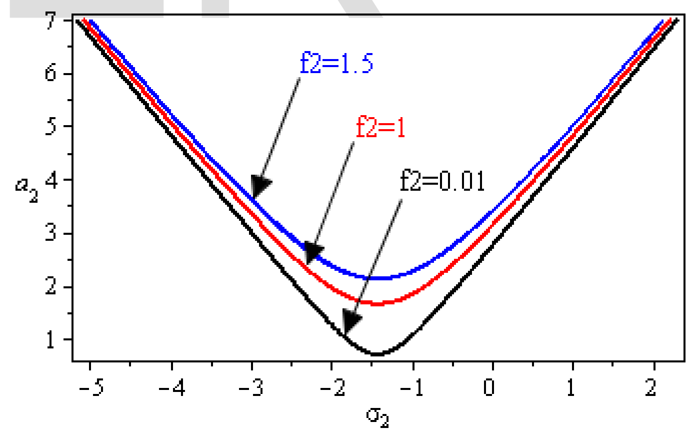

Fig.5d. Effects of the excitation amplitude F_2

5. Comparison Study

In the previous work [14], studied the system of vibro-impact when subjected to external excitation forces. The Poincare´ map of the system is established, and the symmetric fixed point of the Poincare´ map corresponds to the associated symmetric period n-2 motion. It is shown that the Poincare´

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map exhibits some symmetry property, and can be expressed as the second iteration of another unsymmetric implicit map.

In our study, the response and stability of the system of three-degree-of freedom under parametric excitation forces are investigated using the multiple time scale method. All possible resonance cases are extracted and investigated. The case of simultaneous principle parametric resonance in the presence of 1:3 internal resonances is considered. The stability of the system is investigated using both frequency response equations and phase-plane method. It is quite clear that some of the simultaneous resonance cases are undesirable in the design of such system as they represent some of the worst behavior of the system.

5. Conclusions

The nonlinear responses of a vibro-impact system subjected to parametric excitations have been studied. The problem is described by a three-degree-of-freedom system of nonlinear ordinary differential equations. The case of simultaneous principle parametric resonance in the presence of one-to-three internal resonance is studied by applying multiple time scale perturbation method. Both the frequency response equations and the phase-plane technique are applied to study the stability of the system. The effect of the different parameters of the system is studied numerically. From the above study the following may be concluded:

1- The simultaneous resonance case $\Omega \cong 2\omega_2$, $\Omega \cong 2\omega_3$ and $\omega_1 \cong 3\omega_2$ is the worst cases and it should be avoided in design.

2- The steady state amplitude of the first modes of vibro-impact system is a monotonic decreasingfunction in the linear damping coefficients c1.

3- For negative and postive values of the nonlinear parameter α_1 the curves of the first modes of vibro-impact system are bent to the right and left and there exist jump phenomena and multi-valued amplitudes.

4- The steady state amplitude is a monotonic increasing function in the excitation amplitude F1.

5- The steady state amplitude of the second modes of vibro-impact system is a monotonic decreasing function in the linear damping coefficients c2 and the nonlinear parameters $\alpha 2$.

6- The steady state amplitude of the second modes of vibro-impact system is a monotonic increasing function in the excitation amplitude F2.

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